Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1 AND 2
Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

(a) Solve the equation $4(5 - 2x) = x - 7$ for x. (2 marks)

(b) Ash, Billie and Chris collected a total of 160 cans to recycle. Ash collected twice as many cans as Chris, Chris collected 12 more cans than Billie and Billie collected x cans.

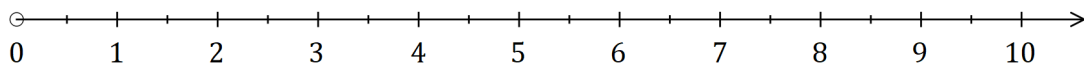Determine how many cans Billie collected. (3 marks)

Question 2 **(5 marks)**

The number of daily absentees at a small school over 15 consecutive days are listed below:

$$7, 6, 4, 5, 6, 6, 3, 9, 7, 6, 7, 6, 4, 5, 4.$$

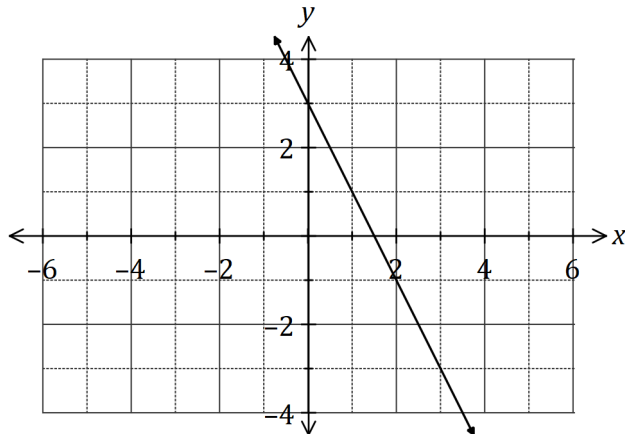(a)   Determine the five-number summary for this data.   (3 marks)

(b)   Use the five-number summary to construct a box-plot on the scale below.   (2 marks)
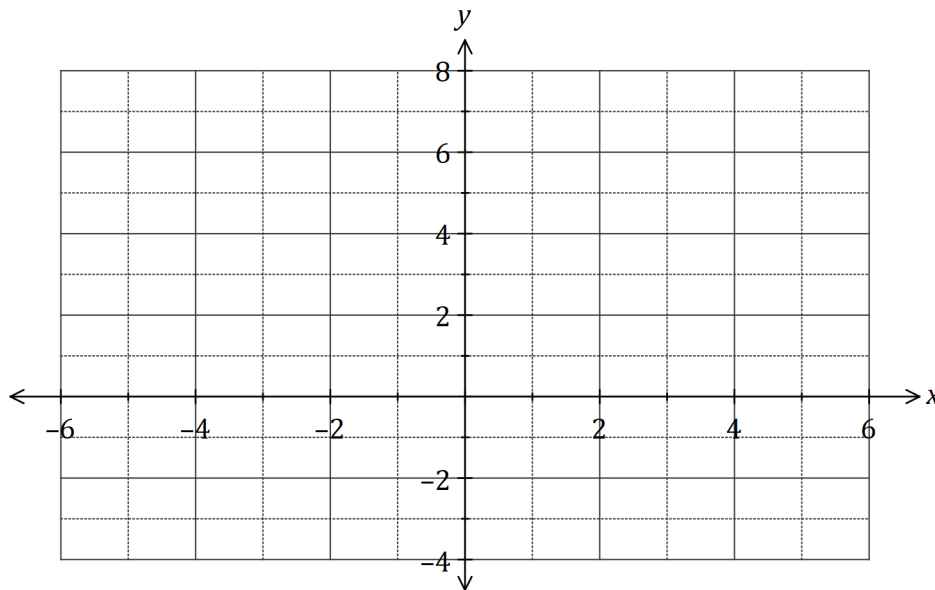
Question 3   **(7 marks)**

(a)   The graph of $y = ax + b$ is shown below. Determine the values of a and b.   (2 marks)

(b)   Draw the graph of the line $y = \dfrac{1}{2}x + 2$ on the axes below.   (3 marks)

(c)   Determine the gradient of the line $3x - 2y = 12$.   (2 marks)

Question 4   **(8 marks)**

(a)   The power P consumed by a device can be calculated using the formula $P = I^2 R + \sqrt{2R}$.

Calculate the value of P when

(i) $I = 5$ and $R = 8$.   (2 marks)

(ii) $I = 0.5$ and $R = 50$.   (2 marks)

(b)   The variable D is related to diameters d_1, d_2 and d_3 by the formula $D = \dfrac{d_1}{4} + \dfrac{2}{d_2 - d_3}$.
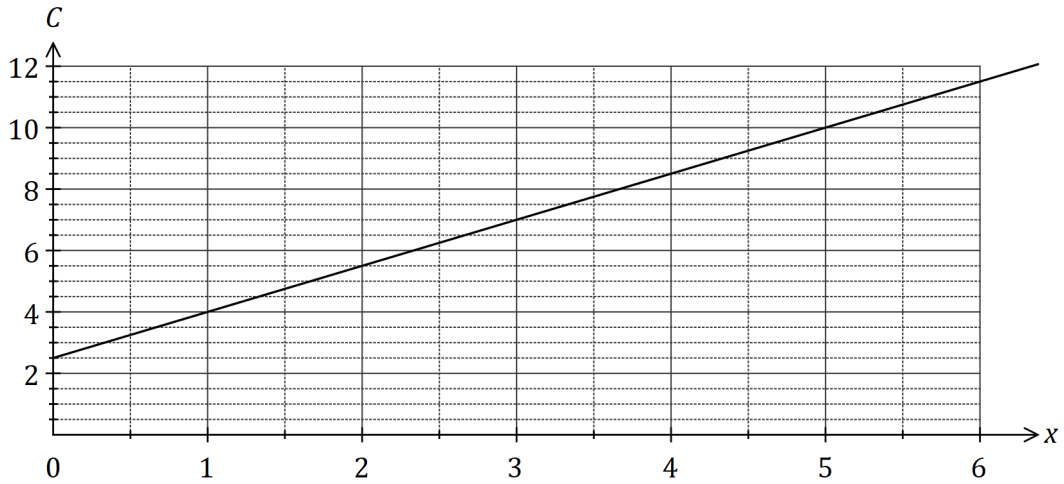
Calculate the value of D when

(i) $d_1 = 2$, $d_2 = 9$ and $d_3 = 1$.   (2 marks)

(ii) $d_1 = 18$, $d_2 = 3.5$ and $d_3 = 4$.   (2 marks)

Question 5 **(6 marks)**

The graph below shows the cost C, in dollars, of taking a journey of x km with a taxi company.

(a)   State the cost of taking a 4 km journey. (1 mark)

(b)   How much more expensive is a 5 km journey compared to one of 4 km? (1 mark)

(c)   State and interpret, in context, the value of the vertical axis intercept of the graph.
(2 marks)

(d)   State and interpret, in context, the value of the gradient of the graph. (2 marks)

Question 6   **(6 marks)**

The heights of 5-year old boys are normally distributed with a mean of 109 cm and a standard deviation of 4 cm.

(a)   Use the 68%, 95%, 99.7% rule to determine the approximate percentage of 5-year old boys that can be expected to have heights between

       (i)   97 and 121 cm.   (2 marks)

       (ii)   109 and 113 cm.   (2 marks)

(b)   Nurses at a health clinic measure the heights of children and refer anyone with a height that is more than two standard deviations from the mean of the child's age group to a doctor.

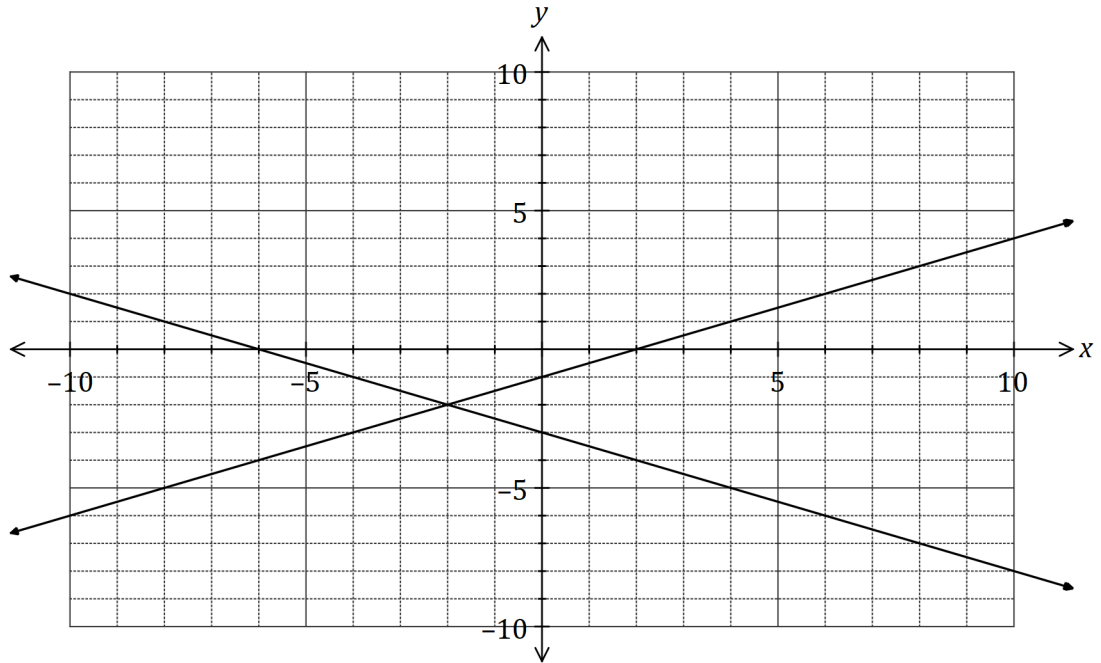   The last eight boys, aged 5, who attended the clinic had the following heights:

$$111 \quad 108 \quad 104 \quad 118 \quad 112 \quad 103 \quad 116 \quad 109 \text{ cm.}$$

      Determine, with justification, how many of these boys were referred.   (2 marks)

Question 7 **(7 marks)**

(a) The lines with equations $2y + 2 = x$ and $2y + x = -6$ are shown on the axes below.

(i) Draw the line $y = x - 6$ on the axes above. (2 marks)

(ii) Solve the simultaneous equations $y = x - 6$ and $2y + 2 = x$. (2 marks)

(b) Solve the simultaneous equations $x + y = 7$ and $2y + x = -6$. (3 marks)

Question 8 **(8 marks)**

A group of friends who frequently travelled abroad exchanged foreign currency between themselves using the conversion table below. For example, members of the group could exchange 100 dollars for 20 dinars or 50 kroner for 10 dollars.

Country/Currency	Australian/Dollar			
	10	20	50	100
Denmark/Krone	50	100	250	500
Japan/Yen	800	1 600	4 000	8 000
Kuwait/Dinar	2	4	10	20
Thailand/Baht	250	500	1 250	2 500

(a) How many baht can be exchanged for 30 dollars? (1 mark)

(b) How many dollars can be exchanged for 20 000 yen? (2 marks)

(c) Before travelling to Kuwait, one of the group exchanged 490 dollars for the local currency. How many dinars did they receive? (2 marks)

(d) Another member of the group exchanged 500 dollars for a mixture of 1 350 kroner and some yen. Determine the amount of Japanese currency they received. (3 marks)

End of questions

Swan
Christian College

MATHEMATICS
APPLICATIONS
UNITS 1 AND 2
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
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Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

The following matrix S shows the number of small sheds (in row 1) and large sheds (in row 2) sold by a company in each of three consecutive months. For example, the element S_{13} represents the number of small sheds sold during the third month.

$$S = \begin{bmatrix} 72 & 68 & 75 \\ 51 & 59 & 38 \end{bmatrix}$$

(a) How many large sheds were sold in the second month? (1 mark)

(b) Calculate matrix A, where $A = S \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (1 mark)

(c) Explain what information matrix A shows. (1 mark)

Matrix $P = [141 \quad 236]$, where P_{11} and P_{12} represent the profit, in dollars, made by selling a small shed and a large shed respectively.

(d) Using matrices A and P, write down a calculation that will result in a matrix showing the total profit from selling all the sheds over the three-month period and state this profit.
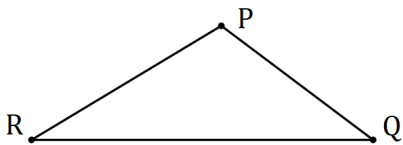(2 marks)

Question 10 **(5 marks)**

(a) Show use of trigonometry to determine the length of side BC in the triangle below, where $\angle BAC = 122°, AB = 58$ cm and $AC = 71$ cm. (2 marks)
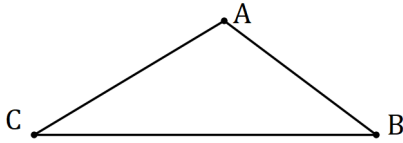
(b) Show use of trigonometry to determine the size of angle $\angle PRQ$ in the triangle below, where $\angle QPR = 105°, PR = 45$ cm and $QR = 65$ cm. (3 marks)

See next page

Question 11 (8 marks)

A hydroponic grower was trialling two different greenhouse systems for growing tomatoes. To compare the systems, the weight of tomatoes produced by each plant in the two greenhouses were recorded. The data is summarised below.
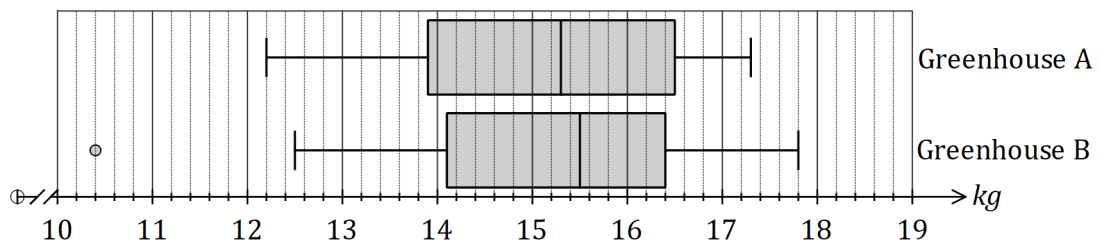
(a) Ignoring the outlier, compare the range of weights produced by plants in greenhouse A with that of greenhouse B. (2 marks)

(b) State and use the interquartile ranges to compare the spread of weights produced by plants in greenhouse A with that of greenhouse B. (2 marks)

(c) Using the result of a relevant calculation, explain why one of the weights in greenhouse B was identified as an outlier. (2 marks)

(d) Explain whether there is evidence to support the conjecture that the system in one greenhouse produces a larger crop of tomatoes than the other. (2 marks)

Question 12 **(7 marks)**

Soup is sold in cylindrical tins that have an internal diameter of 7.6 cm and a height of 13.6 cm.
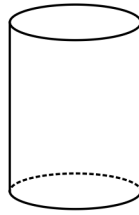
(a) Calculate the internal surface area of the can. (3 marks)

(b) Calculate the capacity of the can in millilitres. (1 mL $= 1$ cm^3) (2 marks)

(c) Before the cans are sealed, they are stood on their circular end and filled with 540 mL of soup. Determine the depth of soup in the can. (2 marks)

Question 13 **(7 marks)**

Individual use coffee bags are packed in boxes of 8, 18 or 28. Customers can buy cartons containing 4, 5 or 6 boxes, as shown in the following table.

Carton	Carton price ($)	Boxes per carton	Coffee bags per box
A	39.20	4	28
B	17.76	6	8
C	30.60	5	18

(a) A customer orders a total of 16 cartons, comprising 7 of type A, 4 of type B and the rest of type C. Calculate the cost of this order, given that orders of more than $150 qualify for a 25% discount. (3 marks)

(b) Determine the cost of one coffee bag in each type of carton and hence list the carton types from best to worst value in terms of the price per coffee bag. (4 marks)

Question 14   **(11 marks)**

As part of an investigation into youth fitness, a researcher collected the sit-and-reach (SR) measurements of 250 students. The data is summarised in the table below.

SR measurement (cm)	Number of students
$18 < x \le 22$	5
$22 < x \le 26$	10
$26 < x \le 30$	32
$30 < x \le 34$	68
$34 < x \le 38$	90
$38 < x \le 42$	36
$42 < x \le 46$	9

(a)   Use the mid-point of each class interval to determine the mean and standard deviation of the sit-and-reach measurements.   **(2 marks)**

(b)   Explain why it was necessary to use the mid-point of each class interval to determine the statistics in (a).   **(1 mark)**

(c)   Draw a histogram on the axes below to display the distribution of SR measurements.

      (3 marks)
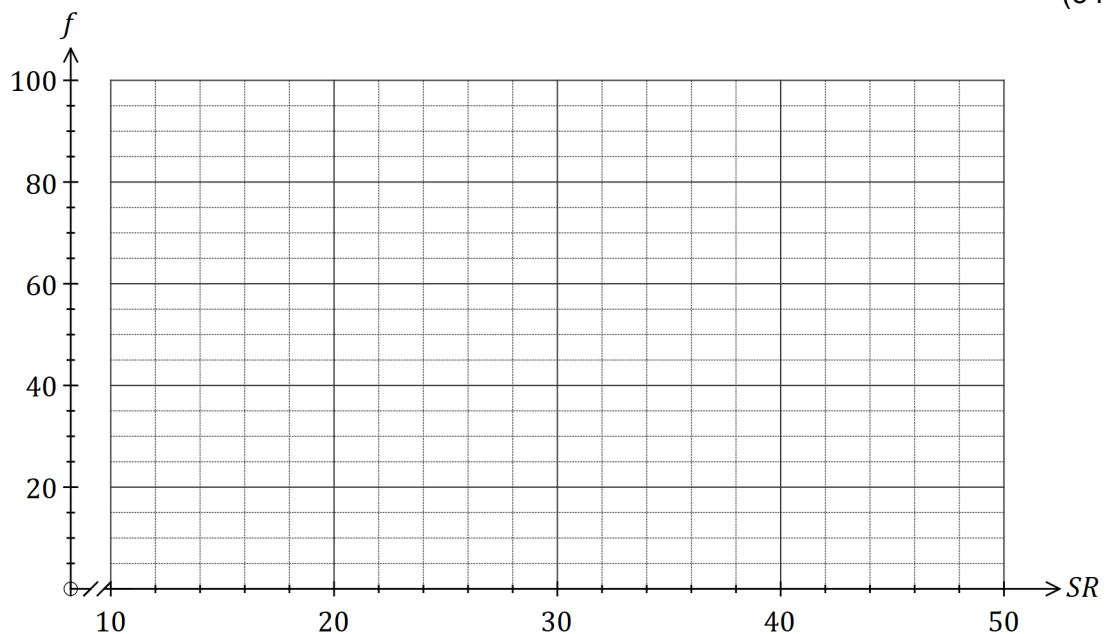
(d) Use features of the histogram to describe the distribution of SR measurements for this group of students. (3 marks)

(e) In a previous investigation, the researcher found that the SR measurements for an older group of people were normally distributed with a mean of $29\,\text{cm}$ and a standard deviation of $3.4\,\text{cm}$. Determine the percentage of people in this older group who had an SR measurement

(i) less than $25\,\text{cm}$. (1 mark)

(ii) between 28 and $38\,\text{cm}$. (1 mark)

Question 15   **(8 marks)**

The roof of a circular building has the shape of a right-circular cone with a base radius of 5.6 m and a perpendicular height of 3.3 m.
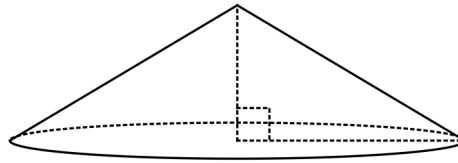
(a)   Building regulations require that one air vent is required for every 30 m^3 of space (or part) in the roof. Determine the number of air vents required for the roof shown.   (3 marks)

(b)   The curved surface of the roof is to be tiled (excluding the circular base). The tile that the builder has chosen has a coverage rate of 16 tiles per square metre and can be bought in packs of 124. Determine the number of packs the builder must order.   (5 marks)

Question 16 **(8 marks)**

(a) The statistical investigation process is a cyclical process that begins with the need to solve a real-world problem and aims to reflect the way statisticians work. Step P is one of the four steps in the cyclical process and involves the design and implementation of a plan to collect or obtain appropriate data.

(i) Describe the key elements of the step immediately after P in the cyclical process.
(2 marks)

(ii) Describe the key elements of the step immediately before P in the cyclical process.
(2 marks)

(b) A student was carrying out a statistical investigation involving dogs.

(i) Describe an example of a categorical variable the student could investigate and list two different responses that could be recorded. (2 marks)

(ii) Describe an example of a continuous numerical variable the student could investigate and list two different responses that could be recorded. (2 marks)

Question 17   **(9 marks)**

A second-hand car yard paid $33 500 for a vehicle and later sold it to a customer for $26 930.

(a)   Calculate the percentage loss made on the sale.   (2 marks)

(b)   The customer paid a deposit of $2 930 and took out a loan for the remainder of the price with an interest rate of 10.9% per annum. Calculate the simple interest on the loan for the first month.   (3 marks)

(c)   The price the customer paid included 10% GST. Calculate the amount of GST included in the price.   (2 marks)

(d)   The standard premium to insure the vehicle was $2 388, but the customer was offered a discount of 35% for not having made any claims over the past five years. Determine the premium after the discount was applied.   (2 marks)

Question 18 **(8 marks)**

The top of a vertical pole T stands 6.4 m above a surrounding level playing field. The angle of depression from T to a small animal at A is $32°$. The animal leaves A, moves directly towards the base of the pole B and stops at C (before reaching B). The distance BC is 5.5 m.

(a)   Sketch a diagram to show the above information. (2 marks)

(b)   Calculate the line of sight distance from the top of the pole to A. (2 marks)

(c)   Determine the angle of depression from T to C. (2 marks)

(d)   Calculate the distance travelled by the animal from A to C. (2 marks)

**Question 19**　　　　　　　　　　　　　　　　　　　　　　**(7 marks)**

The wind chill index I is a measure of how quickly a person exposed to a wind will lose heat. It is calculated using the formula below, where v is the speed of the wind in metres per second and T is the air temperature in degrees Celsius.

$$I = \left(10\sqrt{v} - v + 10.2\right)(34 - T)$$

(a)　　Determine I when the air temperature is $-3°C$ and there is a wind of $12\ \text{m/s}$ blowing.

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(2 marks)

(b)　　Calculate the change in the wind chill index when the air temperature is $-8°C$ and the strength of the wind decreases from $22\ \text{m/s}$ to $8\ \text{m/s}$.　　　　　(3 marks)

(c)　　A person is likely to suffer from frostbite when the wind chill index reaches $1\ 500$. At what temperature will this happen, on a day when the wind has a speed of $25\ \text{m/s}$?　　(2 marks)

Question 20 **(7 marks)**

A model of a triangular gable has measurements shown below, in centimetres.
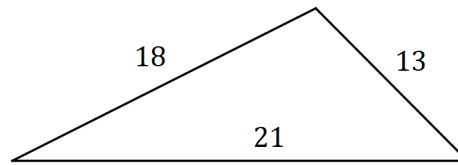
(a) Use Heron's rule to determine the area of the model of the gable. (3 marks)

The model was drawn to a scale using measurements taken from a building, where the length of the shortest side of the gable was 5.2 m.

(b) Calculate the scale factor used to draw the model. (1 mark)

(c) The gable on the building requires repainting, at a cost of $21 per square metre. Determine the cost of repainting the gable, to the nearest dollar. (3 marks)
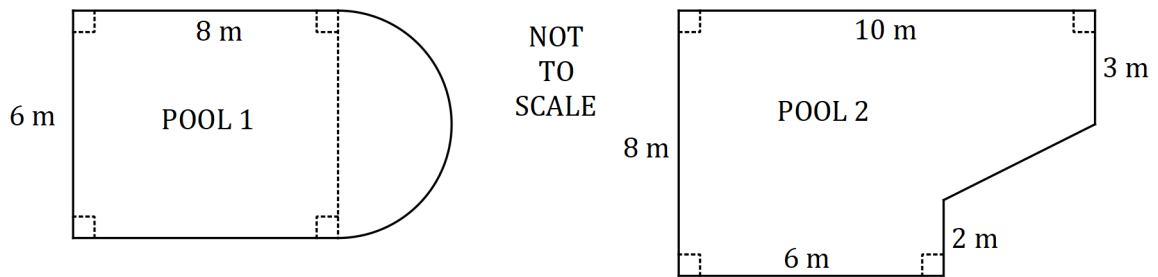
Question 21   **(8 marks)**

When working out the cost of building an in-ground swimming pool, a contractor calculates a shape factor k to use in a spreadsheet, where

$$k = \frac{P^2}{4A}.$$

P is the perimeter of the pool in metres and A is the area of the pool in square metres.

Pool 1 is rectangular with a semi-circular end and pool 2 is rectangular with a cut-out as shown.

Determine, with justification, which of the pools shown has the larger shape factor k.

End of questions